

National Exams May 2015

04-BS-6: Mechanics of Materials

3 hours duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. Candidates may use one of two calculators, the Casio or Sharp approved models.

This is a Closed Book exam. However candidates are permitted to bring the following into the examination room:

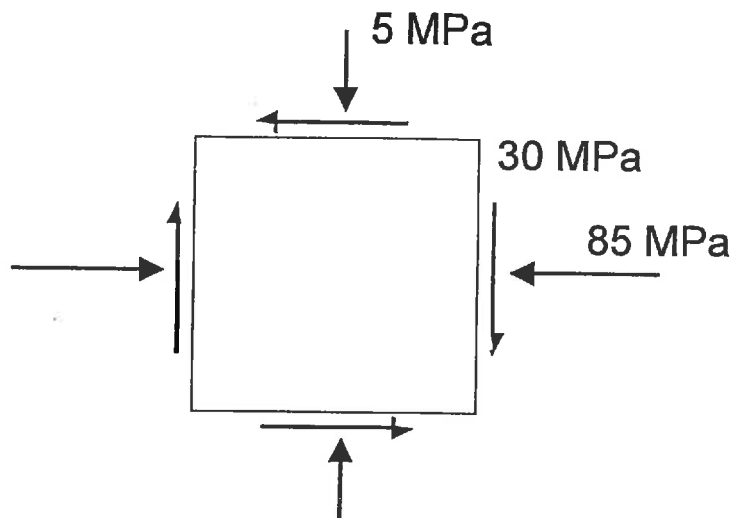
- ONE aid sheet 8.5" x 11" hand-written on both sides containing notes and formulae.
Example problems and solutions to problems are not allowed!
3. Any FIVE (5) questions (out of 8 given) constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
 4. All questions are of equal value.
 5. Information on geometric properties of wide flange or W shape sections is attached at the end of this exam.

NOTE: The aid sheet must be handed in with the exam!

Your exam will not be marked if you do not hand in an aid sheet, unless there is a signed statement by the exam invigilator stating that no aid sheet was used for the exam.

Question 1: For an element in a state of plane stress subjected to the normal and shear stresses shown below, use the Mohr's circle solution (*not* the transformation equations) to determine the following:
[20 marks]

- (a) the principal stresses and orientation of the principal planes, showing your answer on a properly oriented element.
- (b) the maximum in-plane shear stress (and associated normal stresses) and orientation of the corresponding planes. Once again, show your answer on a sketch of a properly oriented element.



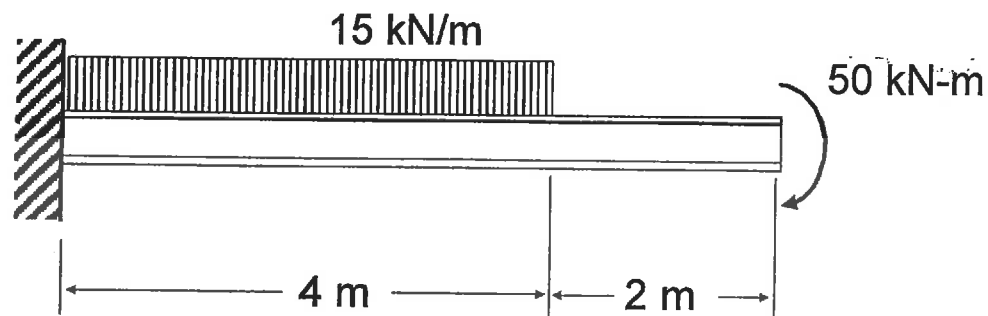
WARNING! Credit will **only** be given for a **solution using Mohr's circle**. Not the stress transformation equations. The stress transformation equations can only be used to check your answer.

This means that you need to draw a Mohr's circle based on the stress components given in this problem. Remember to show numbers on your circle. Your calculations must be based on the geometry of your circle. So use your calculator. In other words, you are expected to use trigonometry to construct your Mohr's circle. Do not give a graphical solution that is scaled off.

Question 2: A cantilevered beam is subjected to a uniformly distributed load over part of the member span in addition to a concentrated couple acting at the free end of the member. The beam is a wide flange W310 x 129 section and is made of steel with an allowable normal stress of 240 MPa and allowable shear stress of 60 MPa. The elastic modulus of the steel equals 200 GPa. Refer to the attached table for section properties.

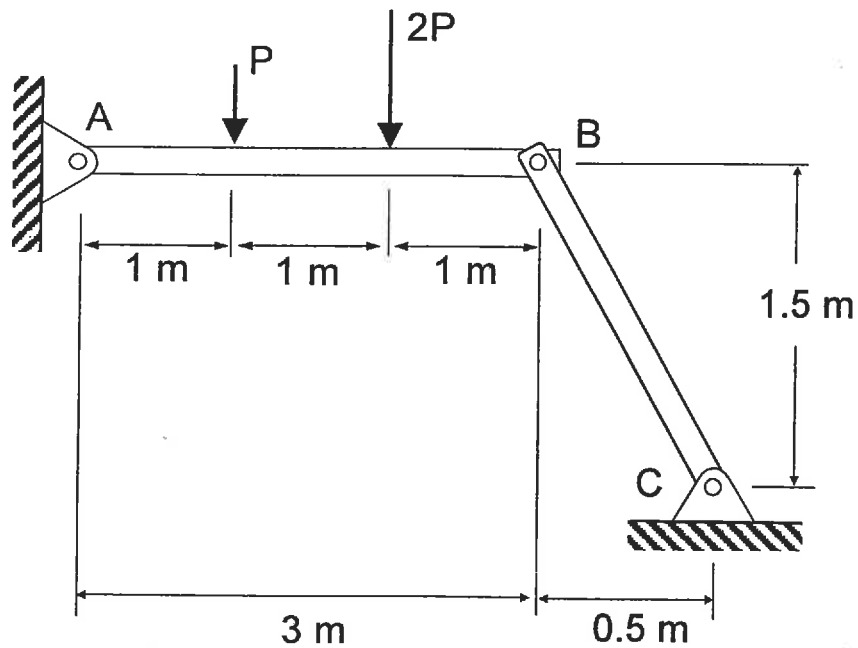
[19 marks] (a) Determine the deflection and slope at the free end of the beam using the method of integration.

[1 mark] (b) Indicate whether the beam satisfies an allowable deflection limit of $L/120$ (where L equals the span of the beam).



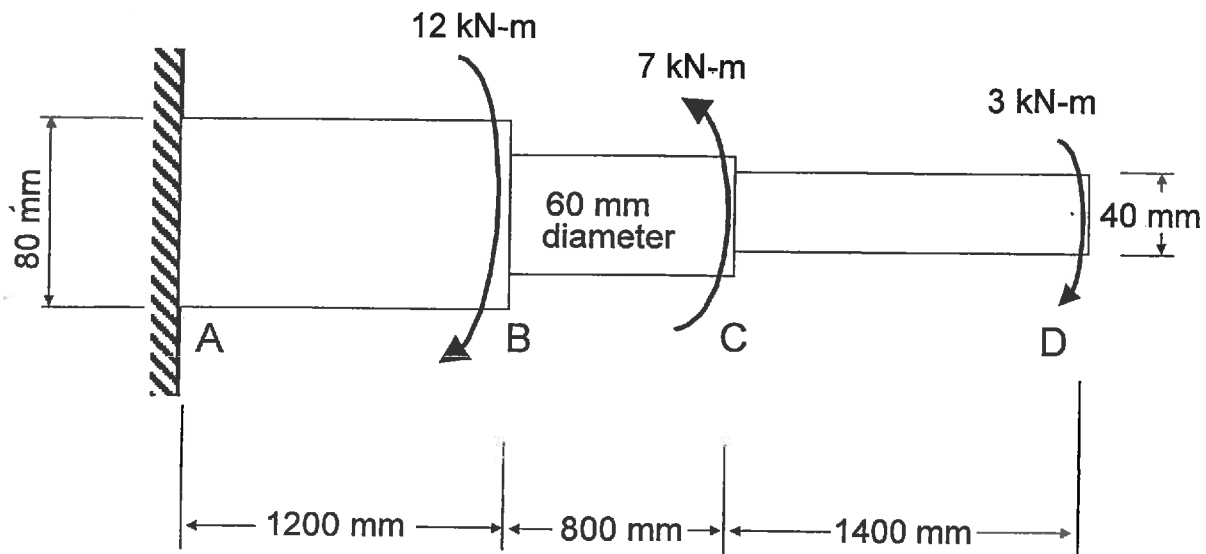
Question 3: A horizontal beam AB is supported by a pinned connection at A and a pin-ended column at B. The column BC has a cross sectional area $40 \text{ mm} \times 40 \text{ mm}$ and is made of steel with an elastic modulus of 200 GPa and allowable yield strength of 280 MPa . The beam is loaded by two concentrated loads as shown.

[20 marks] Determine the largest loads (P and $2P$) that can be applied to the beam without causing the column BC to buckle. Use a safety factor of 2 against buckling and consider buckling in the plane of the structure only. Do not use a safety factor for yielding of the steel, and assume the beam can withstand the applied loads.



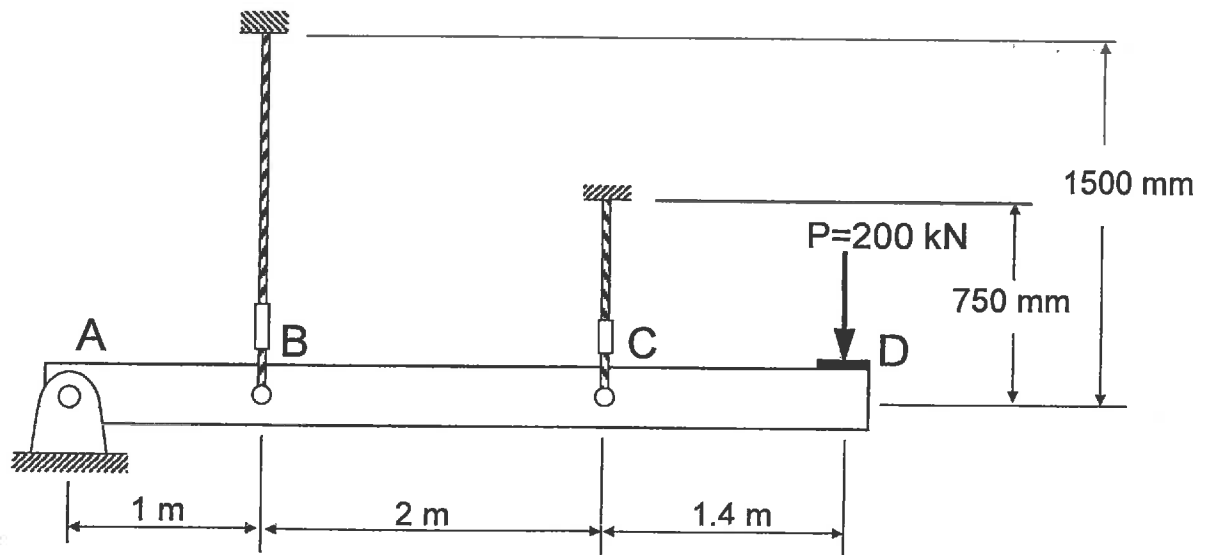
Question 4: A stepped shaft ABCD is subjected to three concentrated torques (acting at B, C and D) as shown. The shaft is fixed at A. The entire shaft is made of high strength steel having a shear modulus $G = 80 \text{ GPa}$ and a yield stress of 250 MPa . Dimensions (diameter and length) and magnitude of the torques are given in the diagram.

- [12 marks] (a) determine the maximum shear stress in the shaft and sketch the variation of shear stress along the shaft radius for the cross-section where the stress is maximum.
- [6 marks] (b) find the angle of twist at the end of the shaft (point D) and give your answer in degrees.
- [2 mark] (c) what would happen if the loads on the shaft were doubled?



Question 5: A rigid beam ABCD is supported by a pin at A and two cables located at B and C. The cables have a 30 mm diameter and are made of high strength steel with a yield strength of 800 MPa and elastic modulus of 200 GPa. The rigid beam is designed to resist a vertical load P applied at D equal to 200 kN.

- [12 marks] (a) find the forces developed in each cable
- [4 marks] (b) find the corresponding vertical displacement at point D where the load is applied
- [4 marks] (c) find the shear stress in the pin at A given that the pin has a diameter of 50 mm and is loaded in double shear

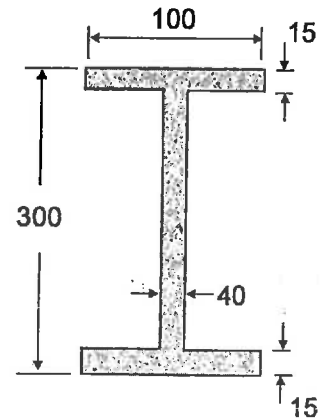
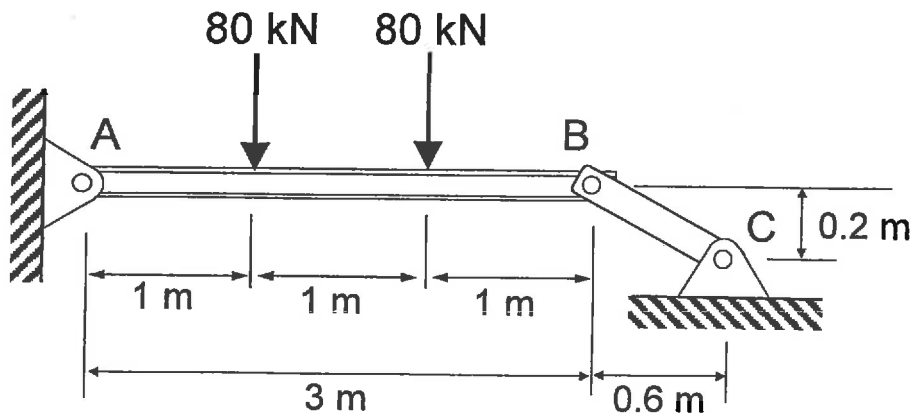


Question 6: A horizontal beam AB is supported as shown by a pinned connection at A and an inclined strut at B. The beam has the cross section shown and is made of steel with a normal yield stress of 350 MPa and yield stress in shear of 60 MPa. The elastic modulus of the steel equals 200 GPa. Assume the strut BC can withstand the applied loads.
 [20 marks]

- (a) Compute the distribution of **normal stress** in the I-beam at a section located **0.7 m from the support at A**. Show this distribution on a sketch and make sure to show maximum and minimum values of stress.
- (b) Compute the maximum **shear stress** in the I-beam at the same section located **0.7 m from the support at A**.

[4 marks
 Bonus]

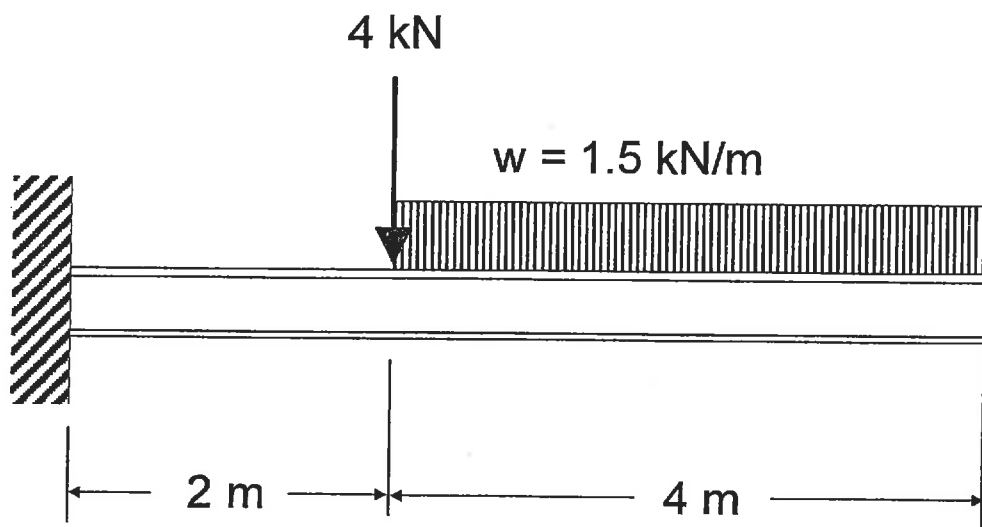
- (c) Determine the plastic (moment) capacity of the section



beam cross-section
 (all dimensions in mm)

Question 7: A cantilevered beam is subjected to a uniformly distributed load over part of the member span in addition to a concentrated load acting 2 m from the fixed support as shown. The beam is made of steel with a yield strength of 350 MPa and shear stress at yield of 75 MPa. The elastic modulus of the steel is 200 GPa. **[20 marks]**

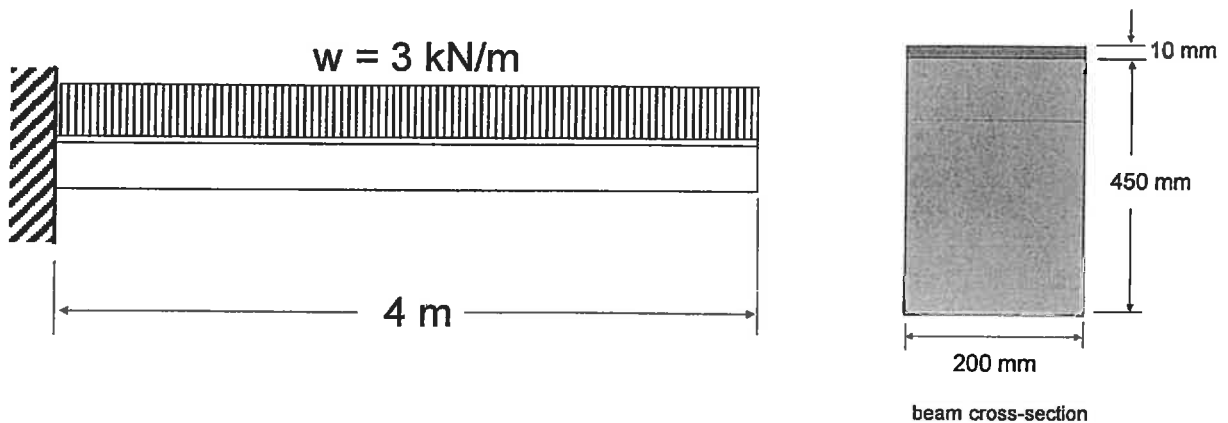
Determine the SHEAR FORCE and BENDING MOMENT along the length of the beam as a function of x . In other words, find $V(x)$ and $M(x)$ for the beam. Then draw the corresponding shear force and bending moment diagrams for the beam (label all critical points and show your work by indicating exactly how you obtained your answers).

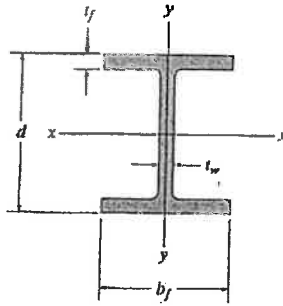


Question 8: The cantilevered beam shown below is a composite member made of a rectangular concrete section (200 mm by 450 mm in cross-section) reinforced with a 10 mm thick steel plate at the top. The member is subjected to the uniformly distributed load shown and bending is about the horizontal axis of the cross-section. The concrete has an allowable normal stress of 3 MPa in tension and 15 MPa in compression, while the steel has an allowable normal stress of 240 MPa. The elastic modulus of the concrete is 30 GPa while that of the steel is 210 GPa.

[18 marks] (a) Determine whether the composite beam can support the loading shown (remember to check for failure in each material)

[2 marks] (b) Give the maximum load w the beam can support without causing failure





Wide-Flange Sections or W Shapes SI Units											
Designation	Area A	Depth d	Web thickness t _w	Flange		x-x axis			y-y axis		
				width b _f	thickness t _f	I	S	r	I	S	r
mm × kg/m	mm ²	mm	mm	mm	mm	10 ⁶ mm ⁴	10 ³ mm ³	mm	10 ⁶ mm ⁴	10 ³ mm ³	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2
W250 × 149	19 000	282	17.30	263.0	28.4	259	1840	117	86.2	656	67.4
W250 × 80	10 200	256	9.40	255.0	15.6	126	984	111	43.1	338	65.0
W250 × 67	8 560	257	8.89	204.0	15.7	104	809	110	22.2	218	50.9
W250 × 58	7 400	252	8.00	203.0	13.5	87.3	693	109	18.8	185	50.4
W250 × 45	5 700	266	7.62	148.0	13.0	71.1	535	112	7.03	95	35.1
W250 × 28	3 620	260	6.35	102.0	10.0	39.9	307	105	1.78	34.9	22.2
W250 × 22	2 850	254	5.84	102.0	6.9	28.8	227	101	1.22	23.9	20.7
W250 × 18	2 280	251	4.83	101.0	5.3	22.5	179	99.3	0.919	18.2	20.1
W200 × 100	12 700	229	14.50	210.0	23.7	113	987	94.3	36.6	349	53.7
W200 × 86	11 000	222	13.00	209.0	20.6	94.7	853	92.8	31.4	300	53.4
W200 × 71	9 100	216	10.20	206.0	17.4	76.6	709	91.7	25.4	247	52.8
W200 × 59	7 580	210	9.14	205.0	14.2	61.2	583	89.9	20.4	199	51.9
W200 × 46	5 890	203	7.24	203.0	11.0	45.5	448	87.9	15.3	151	51.0
W200 × 36	4 570	201	6.22	165.0	10.2	34.4	342	86.8	7.64	92.6	40.9
W200 × 22	2 860	206	6.22	102.0	8.0	20.0	194	83.6	1.42	27.8	22.3
W150 × 37	4 730	162	8.13	154.0	11.6	22.2	274	68.5	7.07	91.8	38.7
W150 × 30	3 790	157	6.60	153.0	9.3	17.1	218	67.2	5.54	72.4	38.2
W150 × 22	2 860	152	5.84	152.0	6.6	12.1	159	65.0	3.87	50.9	36.8
W150 × 24	3 060	160	6.60	102.0	10.3	13.4	168	66.2	1.83	35.9	24.5
W150 × 18	2 290	153	5.84	102.0	7.1	9.19	120	63.3	1.26	24.7	23.5
W150 × 14	1 730	150	4.32	100.0	5.5	6.84	91.2	62.9	0.912	18.2	23.0