

04-CHEM-B1, TRANSPORT PHENOMENA

May 2016

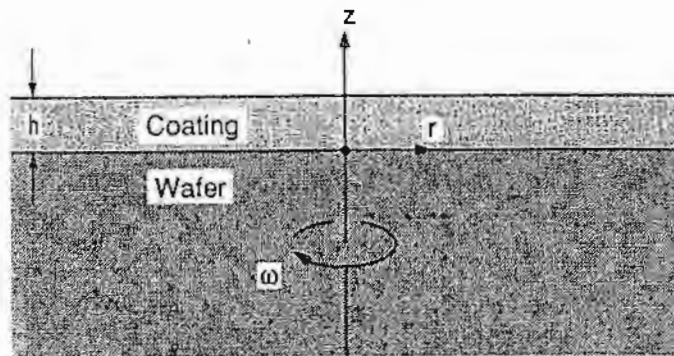
3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. The examination is an **open book exam**. One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.
3. Candidates may use any **non-communicating** calculator.
4. All problems are worth 25 points. **One problem** from **each** of sections A, B, and C must be attempted. A **fourth** problem from **any section** must also be attempted.
5. **Only the first four** questions as they appear in the answer book will be marked.
6. State all assumptions clearly.

Section A: Fluid Mechanics

- A1.** The simplest patient infusion system is that of gravity flow from an intravenous (IV) bag. A 500 ml IV bag containing an aqueous solution is connected to a vein in the forearm of a patient. Venous pressure in the forearm is 0 mm Hg (gage pressure). The IV bag is placed on a stand such that the entrance to the tube leaving the IV bag is exactly 1 meter above the vein into which the IV fluid enters. The length of the IV bag is 30 cm. The IV is fed through an 18-gage tube (internal diameter = 0.953 mm) and the total length of the tube is 2 meters. Calculate the flow rate of the IV fluid, and estimate the time needed to empty the IV bag.
- A2.** The spin coating process is used to coat a thin film of photoresist (organic polymer) on the wafer. A viscous solution containing the photoresist material is made to form a thin ($\sim 1 \mu\text{m}$) uniform thin film on the wafer by spinning the wafer as shown in the figure below.



- a) [22 points] Derive an expression for thickness of the film on the wafer (h) as a function time (t), angular velocity (ω), and the kinematic viscosity (ν) of the photoresist solution.
- b) [3 points] What is the corresponding equation for thickness of a very thin film of photoresist on the wafer?

Section B: Heat Transfer

- B1.** Air at 1 atm and 150 °C enters a pipe of diameter 5.08 cm, and it is heated as it moves through the pipe at a velocity of 8 m/s. Assume constant heat flux per unit length of the pipe, and that the pipe wall temperature is 20 °C above air temperature.
- a) [18 points] Determine the heat transfer per unit length of the pipe.
- b) [7 points] Calculate the bulk temperature rise through a 2-meter length of pipe.

DATA:

Thermal conductivity of air = 3.52×10^{-2} W/m °C
Viscosity of air = 2.38×10^{-5} kg/m.s
Specific heat capacity of air = 1.017 kJ/kg °C
Molecular weight of air = 29 g/mol

- B2.** Derive the differential energy balance equation in spherical coordinates for heating a sphere of radius r , if the temperature variation with angular position is zero.

Section C: Mass Transfer

- C1. For rate of mass transfer from a cylinder rotating in a liquid bath, the Sherwood number (Sh) is given by the equation

$$\text{Sh} = k_d / U_r = 0.079 (\text{Re})^{-0.3} (\text{Sc})^{-0.644}$$

where k_d is the mass transfer coefficient between cylinder and liquid, U_r is the peripheral speed of rotation, Schmidt number $(\text{Sc}) = \nu / D_{AB}$, Reynold's number $(\text{Re}) = dU/\nu$, and kinematic viscosity $(\nu) = \mu/\rho$.

In an experiment, a 1.5-cm diameter by 5.0-cm long iron cylinder was rotated at a speed of 77 rpm in a melt of carbon-saturated iron at 1275 °C. The rate of dissolution, expressed as decrease of rod radius with time, was -5.57×10^{-3} cm/s. For the dissolution of iron in carbon-saturated iron, the iron mass concentration driving force (ΔX_{Fe}) was estimated to be 0.385. Assuming the densities of the iron rod and the carbon-saturated iron melt are equal, compare the experimental rate of dissolution with that predicted by the above correlation.

DATA:

Diffusivity of iron in the carbon-saturated iron melt at 1275 °C = 9×10^{-5} cm²/s

Kinematic viscosity of iron melt at 1275 °C = 1.41×10^{-2} cm²/s

- C2. During class break, a guy comes and cleans the blackboards. He leaves a thin film of water whose thickness is 1 mm. Water evaporates from that surface into the surrounding air, whose humidity is 10%. We want to calculate how long it takes for the water to evaporate completely. Assume a diffusional distance between the water surface and ambient air of 1 cm. At 25 °C, the vapor pressure of water is 0.0313 atm, and the diffusion coefficient of water vapor in air is 0.282 cm²/s.
- [3 points] Define the problem, draw coordinates, and state assumptions.
 - [13 points] Calculate the steady-state flux of water from the surface.
 - [2 points] Obtain an expression to relate the flux to the disappearance of liquid water.
 - [7 points] Calculate the time required for the water film to evaporate and the chalkboard to dry completely.

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \vec{u}) = 0$		(1.1)
Rectangular coordinates (x, y, z)		
$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$		(1.1a)
Cylindrical coordinates (r, θ, z)		
$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$		(1.1b)
Spherical coordinates (r, θ, ϕ)		
$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\rho u_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(\rho u_\phi) = 0$		(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\frac{1}{\rho} \nabla P + \vec{g} + \nu (\nabla^2 \vec{u})$		(A2)
Rectangular coordinates (x, y, z)		
x-component	$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + g_x + \nu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right)$	(A2a)
y-component	$\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + g_y + \nu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right)$	(A2b)
z-component	$\frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right)$	(A2c)

Cylindrical coordinates (r, θ, z)

$$\begin{aligned}
 & \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} \\
 r\text{-component} \quad & = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \quad (A2d)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} \\
 \theta\text{-component} \quad & = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \quad (A2e)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \\
 z\text{-component} \quad & = -\frac{1}{\rho} \frac{\partial P}{\partial z} + g_z + \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \quad (A2f)
 \end{aligned}$$

Spherical coordinates (r, θ, ϕ)

$$\begin{aligned}
 & \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_r}{\partial \phi} - \frac{u_\theta^2}{r} - \frac{u_\phi^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r \\
 r\text{-component} \quad & + \nu \left[\frac{1}{r^2} \frac{\partial^2}{\partial r^2} (r^2 u_r) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} \right] \quad (A2g)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \left(\frac{u_\phi}{r \sin \theta} \right) \frac{\partial u_\theta}{\partial \phi} + \frac{u_r u_\theta}{r} - \frac{u_\phi^2}{r} \cot \theta = -\frac{1}{\rho r} \frac{\partial P}{\partial \theta} + g_\theta \\
 \theta\text{-component} \quad & + \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\theta}{\partial \phi^2} \right. \\
 & \left. + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \right] \quad (A2h)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi}{r} \cot \theta = -\frac{1}{\rho r \sin \theta} \frac{\partial P}{\partial \phi} \\
 \phi\text{-component} \quad & + g_\phi + \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (u_\phi \sin \theta) \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2} \right. \\
 & \left. + \frac{2}{r^2 \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{2 \cot \theta}{r^2 \sin \theta} \frac{\partial u_\theta}{\partial \phi} \right] \quad (A2i)
 \end{aligned}$$

Table A.3 The Energy Equation for Incompressible Media

$\rho c_p \left[\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla)(T) \right] = [\nabla \cdot k \nabla T] + \dot{T}_G \quad (A3)$	
Rectangular coordinates (x, y, z)	$\rho c_p \left[\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{T}_G \quad (A3a)$
Cylindrical coordinates (r, θ, z)	$\rho c_p \left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} \right] = \frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{T}_G \quad (A3b)$
Spherical coordinates (r, θ, ϕ)	$\rho c_p \left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right] =$ $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{T}_G \quad (A3c)$

Table A4: The continuity equation for species A in terms of the molar flux

$\frac{\partial C_A}{\partial t} = -(\nabla \cdot \vec{N}_A) + \dot{R}_{A,G} \quad (4)$	
Rectangular coordinates (x, y, z)	$\frac{\partial C_A}{\partial t} = - \left(\frac{\partial [N_A]_x}{\partial x} + \frac{\partial [N_A]_y}{\partial y} + \frac{\partial [N_A]_z}{\partial z} \right) + \dot{R}_{A,G} \quad (4a)$
Cylindrical coordinates (r, θ, z)	$\frac{\partial C_A}{\partial t} = - \left\{ \frac{1}{r} \frac{\partial}{\partial r} [r N_A]_r + \frac{1}{r} \frac{\partial}{\partial \theta} [N_A]_\theta + \frac{\partial}{\partial z} [N_A]_z \right\} + \dot{R}_{A,G} \quad (4b)$
Spherical coordinates (r, θ, ϕ)	$\frac{\partial C_A}{\partial t} = - \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 [N_A]_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} ([N_A]_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} [N_A]_\phi \right\} + \dot{R}_{A,G} \quad (4c)$

Table A.5: The continuity equation for species A

$\frac{\partial C_A}{\partial t} + (\vec{u} \cdot \nabla) C_A = D_A \nabla^2 C_A + \dot{R}_{A,G} \quad (5)$	
Rectangular coordinates (x, y, z)	
$\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = \frac{\partial}{\partial x} \left(D \frac{\partial C_A}{\partial x} \right) + \frac{\partial}{\partial y} \left(D \frac{\partial C_A}{\partial y} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G} \quad (5a)$	
Cylindrical coordinates (r, θ, z)	
$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + u_z \frac{\partial C_A}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(D \frac{\partial C_A}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial C_A}{\partial z} \right) + \dot{R}_{A,G} \quad (5b)$	
Spherical coordinates (r, θ, ϕ)	
$\frac{\partial C_A}{\partial t} + u_r \frac{\partial C_A}{\partial r} + \frac{u_\theta}{r} \frac{\partial C_A}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial C_A}{\partial \phi} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(D \sin \theta \frac{\partial C_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(D \frac{\partial C_A}{\partial \phi} \right) + \dot{R}_{A,G} \quad (5c)$	